Introduction

Tensor completion can serve plentiful applications:

- Image inpainting
- Video inpainting
- Hyperspectral recovery
- MRI data recovery
- Context-aware recommendation
- Among others

Motivations

- A few low-rank tensor completion (LRTC) methods have been developed. Roughly speaking, they are classified as unfolding formulation and decomposition formulation.
- Visual data often contains piecewise smooth priors in spatial dimensions, because of edges and objects therein.
- However, the priors are not exploited in existing LRTC methods.
- We propose to combine LRTC with total variation (TV) for incorporating the priors. Better recovery performances are expected, because low-rankness can exploit regular global patterns, and TV can leverage the local piecewise smooth priors.

Objectives

- To develop methods that can nicely combine LRTC and TV together:
  - Unfolding formulation: LRTC_TV_I
  - Decomposition formulation: LRTC_TV_II
  - Compare their performance against state-of-the-art tensor completion competitors
- Our codes are available at https://sites.google.com/site/xutaoli2014

LRTC_TV_I (Unfolding Formulation)

Objective Function (convex)

\[ \min \lambda \sum_{\alpha} \| \mathcal{A}_\alpha \|_1 + \frac{1}{N} \sum_{\alpha} \| \mathbf{Z}_\alpha \|_2, \quad \text{s.t.} \quad [\mathbf{Z}_\alpha]_{ij} = [\mathbf{X}]_{ij} \]

ADMM Optimization of LRTC_TV_I

Main idea: Introduce auxiliary variables to make optimization separable and then apply ADMM.

Datasets

1. Eight color images (256-by-256-by-3 tensor)
2. MRI medical data BRAINIX (288-by-288-by-22 tensor)

Experimental Results (1)

(a) RSE Metric

(b) PSNR Metric

Results on color image inpainting.

LRTC-TV-II (Decomposition Formulation)

Objective Function (non-convex)

\[ \min \lambda_1 \sum_{\alpha} \| \mathcal{A}_\alpha \|_1 + \frac{1}{N} \sum_{\alpha} \| \mathbf{U}^{(\alpha)} \|_1 + \lambda_2 \| \mathbf{Z} \|_2, \quad \text{s.t.} \quad \mathbf{Z} = \mathbf{C} \times_1 \mathbf{C} \times_2 \mathbf{D} \times_3 \mathbf{F}, \quad [\mathbf{Z}]_{ij} = [\mathbf{X}]_{ij} \]

ADMM Optimization of LRTC-TV_II

Main idea: Introduce auxiliary variables to make optimization separable and then apply ADMM.

Datasets

1. Eight color images (256-by-256-by-3 tensor)
2. MRI medical data BRAINIX (288-by-288-by-22 tensor)

Experimental Results (2)

(a) RSE Metric

(b) PSNR Metric

Results on MRI medical data recovery.

- LRTC-TV-I and LRTC-TV-II outperform state-of-the-art competitors, especially when the missing rate is high. Reason: TV smooth prior is useful.
- LRTC-TV-II (nonconvex formulation) works better than LRTC-TV-I (convex formulation). Reason: Nonconvex recovery formulation often requires fewer observations. (Oymak et al. 2015)
- FBCP delivers competitive performance against LRTC-TV-I on color image inpainting, but performs much worse on medical data recovery. Reason: Larger tensor poses more challenges to its rank estimation for FBCP.